Sem:- II

MJC physics 02 Unit 02

Dr. Usha Kumari
Assistant professor
Maharaja College Ara

Topic

"Resultant of Two S.H.M.s at Right Angles with Frequency Ratio 1:2"

$$x = a \sin \omega t \quad \text{——— (1)}$$
$$y = b \sin(2\omega t + \phi) \quad \text{——— (2)}$$

$$\sin \omega t = \frac{x}{a} \quad \text{and} \quad \cos \omega t = \sqrt{\left(1 - \frac{x^2}{a^2}\right)}$$

$$\frac{y}{b} = \sin(2\omega t + \phi)$$
$$= \sin 2\omega t \cos \phi + \cos 2\omega t \cdot \sin \phi$$
$$= 2 \sin \omega t \cdot \cos \omega t \cdot \cos \phi + (1 - 2\sin^2 \omega t) \sin \phi$$

substituting the values of $\sin \omega t$ and $\cos \omega t$,

$$\frac{y}{b} = 2\frac{x}{a}\sqrt{\left(1 - \frac{x^2}{a^2}\right)} \cdot \cos\phi + \sin\phi - 2\frac{x^2}{a^2}\sin\phi$$

$$\left(\frac{y}{b}-\sin\phi\right)+\frac{2x^2}{a^2}\sin\phi=\frac{2x}{a}\sqrt{\left(1-\frac{x^2}{a^2}\right)}\cdot\cos\phi$$

squaring both sides, we get

$$\left(\frac{y}{b}-\sin\phi\right)^2+\frac{4x^2}{a^4}\sin^2\phi+\frac{4x^2}{a^2}\cdot\sin\phi\left(\frac{y}{b}-\sin\phi\right)$$

$$=\frac{4\cancel{x}^2}{a^2}\left(1-\frac{x^2}{a^2}\right)\cos^2\phi$$

$$=\frac{4x^2}{a^2}\cos^2\phi-\frac{4x^4}{a^4}\cdot\cos^2\phi$$

$$\text{or}\left(\frac{y}{b}-\sin\phi\right)^2+\frac{4x^2y}{a^2b}\sin\phi-\frac{4x^2}{a^2}\sin^2\phi+\frac{4x^4}{a^4}\sin^2\phi$$

$$=\frac{4x^2}{a^2}\cos^2\phi-\frac{4x^4}{a^4}\cos^2\phi$$

$$\text{or, }\left(\frac{y}{b}-\sin\phi\right)^2+\frac{4x^2y}{a^2b}\sin\phi-\frac{4x^2}{a^2}\left(\sin^2\phi+\cos^2\phi\right)$$

$$+\frac{4x^4}{a^4}\left(\sin^2\phi+\cos^2\phi\right)=0$$

or $$\left(\frac{y}{b} - \sin\phi\right)^2 + \frac{4x^2y}{a^2b}\sin\phi - \frac{4x^2}{a^2} + \frac{4x^4}{a^4} = 0 \quad \text{--- (3)}$$

Equation (3) gives resultant motion of the particle

Some important cases:-

Case I when $\phi = 0^c$

$$\left(\frac{y}{b} - \sin 0^c\right)^2 + \frac{4x^2y}{a^2b}\sin 0^c - \frac{4x^2}{a^2} + \frac{4x^4}{a^4} =$$

$$\frac{y^2}{b^2} - \frac{4x^2}{a^2} + \frac{4x^2}{a^4} = 0$$

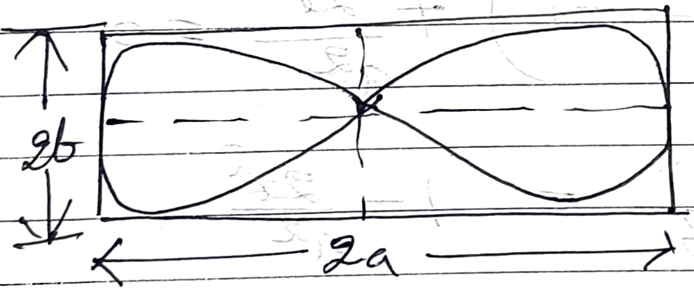

fig (1)

Case :- 2 when $\theta = \left(\frac{\pi}{2}\right)^c$

$$\left(\frac{y}{b} - \sin\frac{\pi}{2}\right)^2 + \frac{4x^2y}{a^2b}\sin\frac{\pi}{2} - \frac{4x^2}{a^2} + \frac{4x^4}{a^4} = 0$$

$$\left(\frac{y}{b}-1\right)^2 + \frac{4x^2y}{a^2b} - \frac{4x^2}{a^2} + \frac{4x^4}{a^4} = 0$$

$$\left(\frac{y}{b}-1\right)^2 + \frac{4x^2}{a^2}\left(\frac{y}{b}-1\right) + \frac{4x^4}{a^4} = 0$$

$$\left[\left(\frac{y}{b}-1\right) + \frac{2x^2}{a^2}\right]^2 = 0$$

$$\left(\frac{y}{b}-1\right) + \frac{2x^2}{a^2} = 0$$

$$\frac{y}{b} = 1 - \frac{2x^2}{a^2}$$

$$\frac{y}{b} = \frac{a^2 - 2x^2}{a^2}$$

$$\therefore y = -\frac{b}{a^2}\left(2x^2 - a^2\right)$$

This equ^n^ represents a parabola as shown in fig ②.

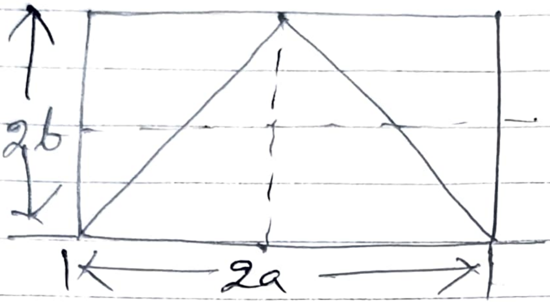

fig ②

The Lissajous's figures in ratio 1:2 are as follows →

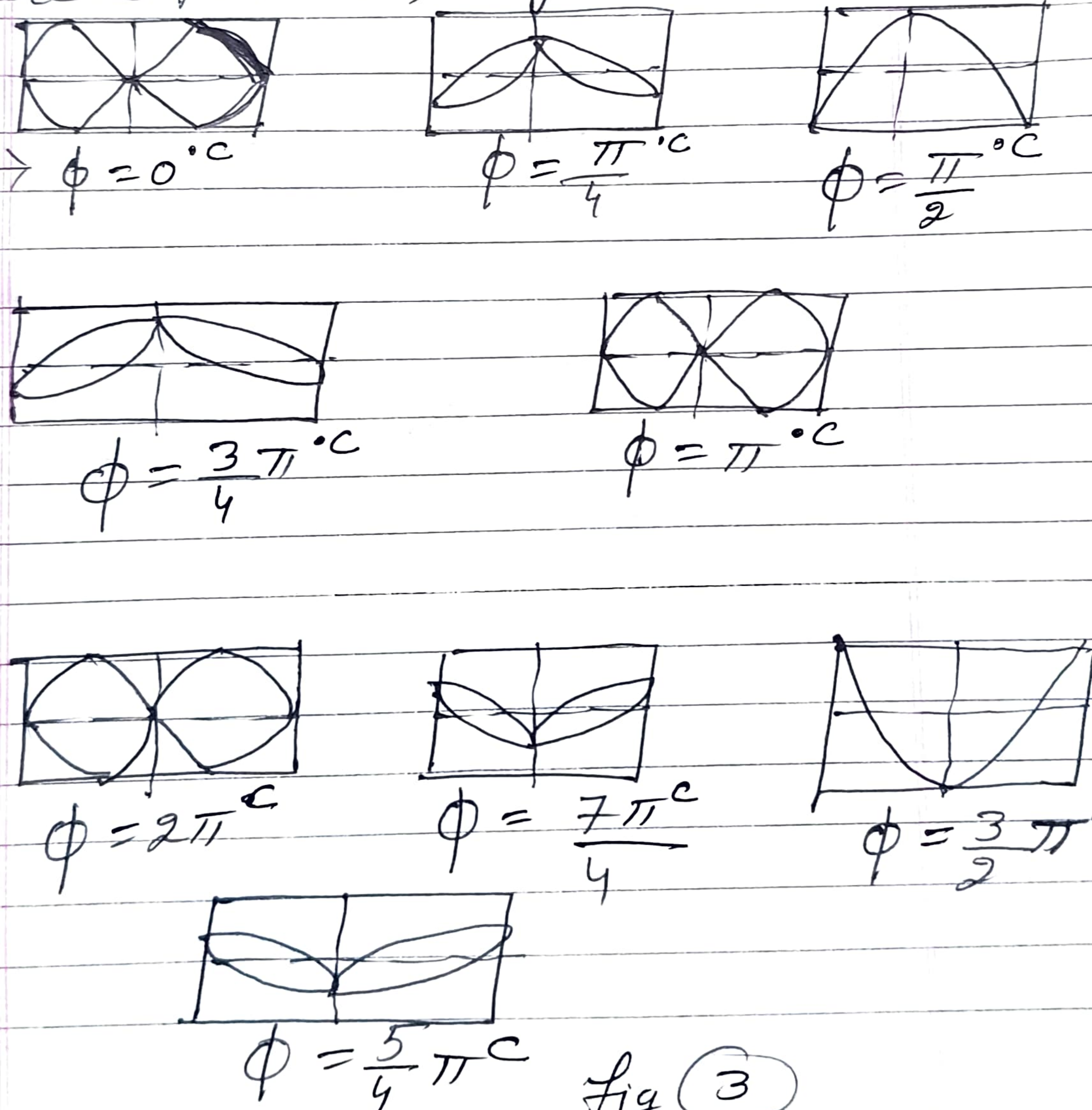

fig ③

—— x ——